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## Introduction

The project is based on the problem of Pebbling a Chessboard, also known as freeing the clones. The original problem involved getting three pebbles out of a specific region of an infinite chessboard by considering it as the first quadrant of the plane. At each move a pebble at the position ( $i, j$ ) was removed and was replaced by the two pebbles at $(i+1, j)$ and ( $\mathrm{i}, \mathrm{j}+1$ ) only if these cells were vacant. This project explores a new dimension to the traditional Pebbling Chessboard game by moulding the problem into a live mobile game and extending the original problem of one quadrant to second and fourth quadrant also. The solvability of the problem has been explored in different quadrants and all the solvable cases has been presented in an interesting way as 405 different levels of the game.. With the simplicity of this game it simply aims to bring fun and make you look to a critical mathematical problem in an attractive way with some new variations

## Technologies

Flutter (Dart) , Figma, Canva, Android Studio

## Game Overview

- The primary objective of the player is to free the pebbles from the pre-defined region known as prison. The player can only win the game if there is no pebble present in the prison
- In the one quadrant game, the player is allowed to move the pebbles into upward and right direction,
In a two quadrants game, the pebbles can be move upward and right or upward and left.
- In the four quadrant game, the pebbles can be placed in all the four directions i.e. upward and right or upward and left or downward and right or downward and left.


## Analysis of Original Problem

Consider the upper half plane where the weight assigned to each cell at $(\mathrm{i}, \mathrm{j})=2^{(\mathrm{i}+\mathrm{j})}$
For $n$, where $n € Z, \quad R_{n}=\frac{1}{2^{n}}+\frac{1}{2^{n+1}}+\frac{1}{2^{n+2}}+\ldots . .+\frac{1}{2^{\infty}}=\frac{1}{2^{n-1}}$
Therefore, the weight of the complete board is $W_{\text {board }}=\sum_{n=0}^{\infty} R n=4$
we denote:
$\mathrm{W}_{\mathrm{T}}$ : The total weight of the chessboard
$W_{\mathrm{O}}$ : Total weight of the cell occupied by the pebbles $W_{R}$ : Weight of the region to be escaped
$W_{A}=W_{T}-W_{R}$ : Weight of the available region
Further, as there are two moves possible in each quadrant,
the degree of freedom of each one of them is two.

| No. of Quadrants | Total weight of te Board | $r$ |
| :---: | :---: | :---: |
| One | 4 | $1 / 3$ |
| Two | 6 | $4 / 5$ |
| Three | 8 | 67 |
| Four | 9 | 1 |

Therefore, the total degree of freedom in a game is

$$
d=2 \times \text { the number of quadrants }
$$

For any game we now define a constant, $\quad \gamma=\frac{d}{w_{T}-1}$

## Methodology

A region is called inescapable if one cannot empty the region of the chessboard by employing any finite sequence of moves.
The possible escapable region in the chessboard can be defined as:
$\mathrm{W}_{T}-\mathrm{W}_{O}-\gamma \leq \mathrm{W}_{R}<\mathrm{W}_{T}-\mathrm{W}_{O}$
Here, $W_{T}=$ Total weight of the chessboard $W_{0}=$ Total weight of the cell occupied by pebbles $W_{R}=$ Weight of the region to be escaped
$r=\frac{d}{W_{T}-1}$
The conjecture stated above is used to find the possible escapable region for different configuration in all quadrants.

Configuration of Game Board in different quadrants


## Weight of cells

- The weight of the cell placed at origin ( 0,0 ) is marked as 1
- At any move, the pebble gets split into two
- Therefore the cells adjacent to origin will have weight $1 / 2$.
- As the pebbles gets split further each time the weight of the cells get reduced by $1 / 2$.


## Movement of pebbles in different quadrants

- There are four possible directions in which the player can move the pebbles in the four quadrants game and in two .directions two quadrants game The directions are Up and right (UR) or Up and left (UL) or down and left (DL) or down and right (DR).
The player can easily change the direction of pebbles by clicking the arrow button present at the bottom center of the screen
If the button is currently showing arrows in upward and right direction then the movement of pebble will be UR, if the arrows are in upward and left direction then movement of pebble will be UL, if the arrows are in downward and left direction then movement of pebble will be UL, if the arrows
movement of pebble will DL and DR otherwise
$\leftrightarrow \downarrow$


## Conclusion

In this game, we have implemented a primitive mathematical puzzle into an interesting mobile game with interactive User Interface and easy accessibility.
This project is a good start towards the portability of mobile games and in turn the mobile applications and reflects the application of mathematics in the modern technology.

## Future Work

1. Building an algorithm which will automatically select prison size and levels.
2. Adding more levels in the four quadrants game.
3. Comparison of the ability of the player with the minimum number of moves

