Analysing inherent beauty of traditional Kolam art with a Mathematical lens

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Introduction

Kolam is a creative creation. It is a ubiquitous art form most important in South India, at the same time as also visible in a few locations in northern India and South East Asia.Kolams are generated using kolam grammar. We can generate many kolams with a variety of pullis (dots) with a finite quantity of regulations Kolam drawing may be dealt with as a unique sort of a graph with the crossings taken into consideration as vertices and the parts of the kambi between vertices dealt with as edges. The only limit is that unlike in a graph, these edges cannot be freely drawn as there is a selected way of drawing the kolam. The single kambi kolam will then be an Eulerian graph with the drawing starting and finishing inside the identical vertex and passing through every fringe of the graph simplest once.

Basic Units for Kolam

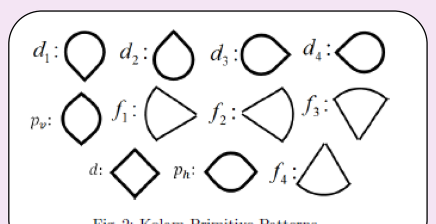
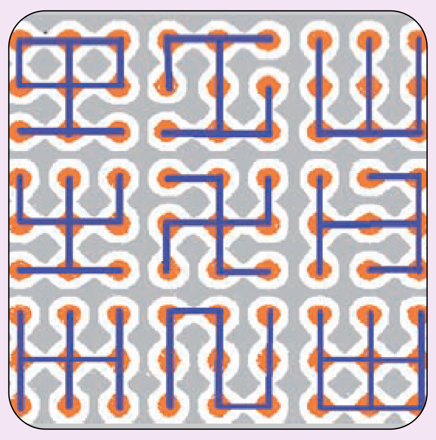


Fig. 2: Kolam Primitive Patterns.



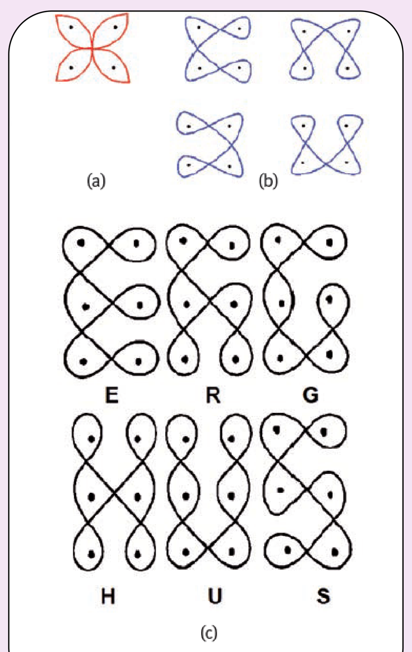


Figure 11 The 2 × 2 and 2 × 3 kolams - building blocks of the Fibonacci kolams.

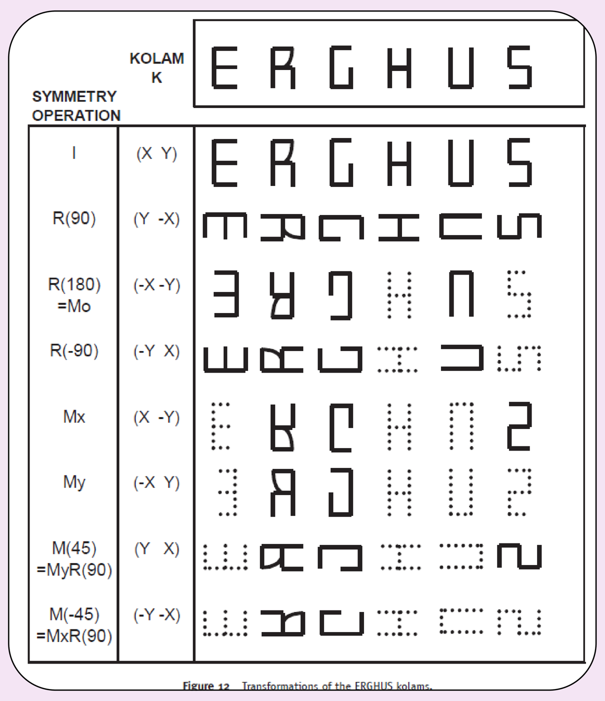


Figure 12 Transformation of the ERGHUS kolams.

Rules for Splicing/Unsplicing

To make a kolam,connect all basic elements to form one singular pattern. The process of joining them is called splicing. Following are the rules/outcome of the splicing-
→ a splice between two loops gives one loop
→a splice within a single loop splits it into two loops
In practice a good strategy to obtain maximal splicing consistent with a single loop is the following. Splice all allowed splicing points at one 'go'. If the number of loops is one, the task is done. If the number is more than one, then unsplice a set of four points to make the kolam single loop; the choice of this set will require some 'trial and error' experimentation. As a corollary to the 'splicing rules', the 'un-splicing rules' are:
→un-splicing at the intersection of two loops give one loop
→un-splicing within a single loop will split it into two loops.

Designing Single Knot Kolam

Single Knot kolam is also called as Antathi Kolam in Southern India. It is designed in a way that it has a four way rotational symmetry. It is designed by using the basic primitives, which are the basic building blocks for this type of kolam. We shall design an algorithm for this type of Kolam.
Define initiator:
→Initiator is the step that will be the base of the kolam. If the iterations are set to zero, then this will be set as default. We take the Initiator as 'FBFBFBFB'
Set a extension:
→Extension will be added after every iteration to the selected points for further iterations.The most common extension is 'AFBFA'
Finalise with Corners.
→Corners will be added to every iteration for completion of Kolam.For this kolam, we define Corner as 'AFBFBFBFA'
Selection of points for additions of elements
The following algorithm is used for adding elements to the pattern:

```
for i in range(n):
    for ch in initiator:
        if ch == F → pattern =+ F
        if ch == B → pattern =+ corner
        if ch == A → pattern =+ extension
    initiator = pattern
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The following are the results for this algorithm

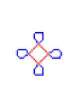


Fig- 0th Iteration

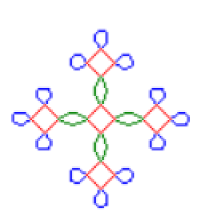


Fig-1st Iteration

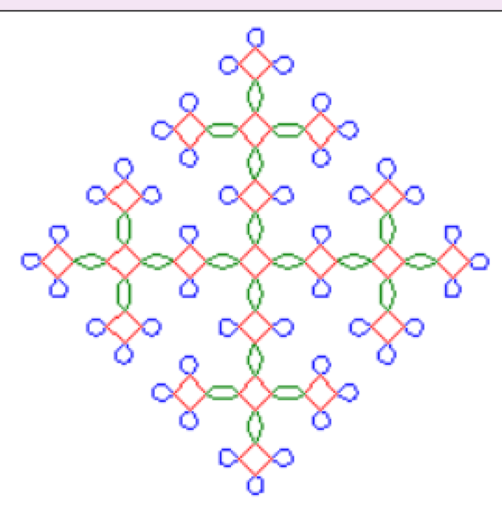


Fig- 2nd Iteration

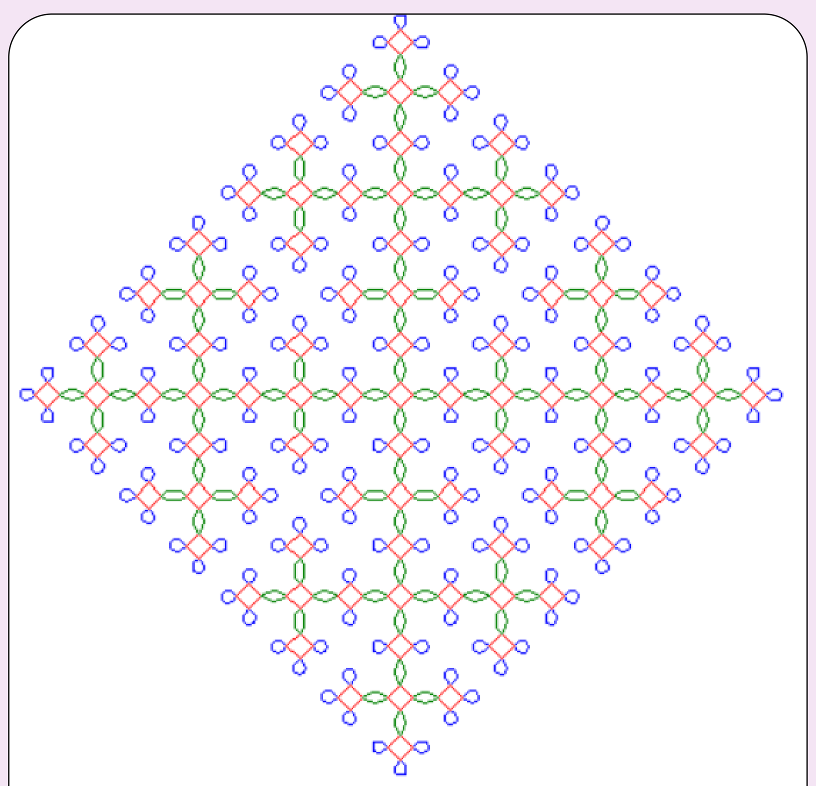


Fig- 3rd Iteration

This tpye of Kolam mathematically falls under the catagory of Regular Matrix Kollam. To this class belong all the kolam patterns where the dots (pulli) form m × n rectangular arrays, m,n>1. This category consists of kolams that can be Eulerian or non-Eulerian. The pattern is recursive here, so they can be expanded using certain steps. Since in this case, we were able to draw the kolam in one go, this is an Eulerian Finite Matrix Kolam

Result and Future Work

→Kolam are classified in 2 main types that is Finite Matrix and Regular Matrix.
→ Regular Matrix Kolam have 4 way symmetry, commonly known as Antathi Kolam. They can be expanded iteratively applying a certain algorithm.
→ Finite Matrix Kolam have 2 way rotaional or mirror symmetry, commonly known as Pulli Kolam. They have no certain pattern, hene cannot be expanded iteratively.
→Mathematical Kolam can be formed with any of the mathematical constant whose either fractional form or continued fraction array is known like pi, e etc.
→Future work for this project will be to generate a Kolam for any mathematical constant enterd by the user.

Quartet Rules for Square/Non-Square Kolam

For kolam designs we would consider a set of four consecutive integers in a generalised Fibonacci series such as (3 5 8 13), (3 4 7 11). In a quartet Q(a b c d) we have

a + b = c
c +b = d

The relation between a,b,c and d can be given by the following equations:

ab + b2 = ac →1
d2 = a2 + 4bc →2

In generating square kolams invariably rectangular kolams appear as constituents. These rectangles are golden rectangles (ratio of sides z = 1.618...), but they lack the two-fold rotational symmetry (appearing the same view from north or south). 'Rectangles' with two-fold symmetry can be built with two Fibonacci quartets. The equations are very similar to that of 1 and 2 -

Q(a1,b1,c1,d1) Q(a2,b2,c2,d2)
a1+ b1 = c1; c1+ b1= d1 a2+ b2 = c2; c2+ b2= d2

Similarly we have
d1d2 = a1a2 +2b1c2 + 2b2c1

These Equations help in determining the skeleton for the kolam pattern we shal make. The following is the example of the quartet division used in Fibonacci series, which is for a squre skeleton designs. For non-Square patterns, we take different constants that are used in mathematics like π and e. The following diagrams show square and rectangle skeletons-

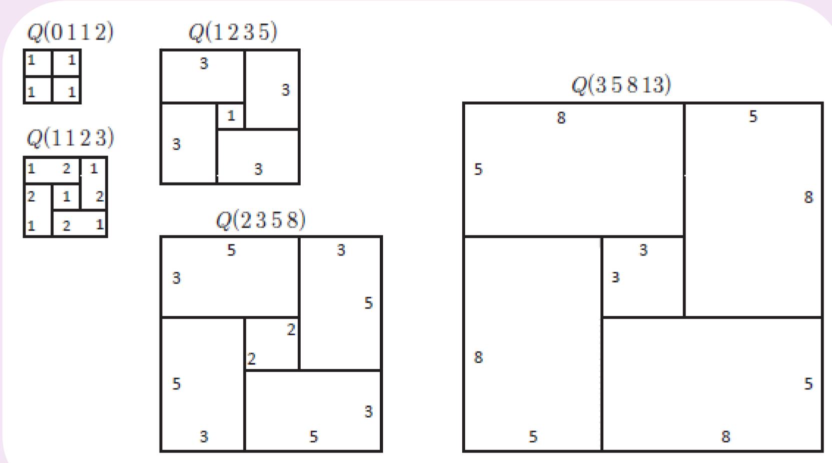


Figure 4 Skeletons of squares for Fibonacci kolams

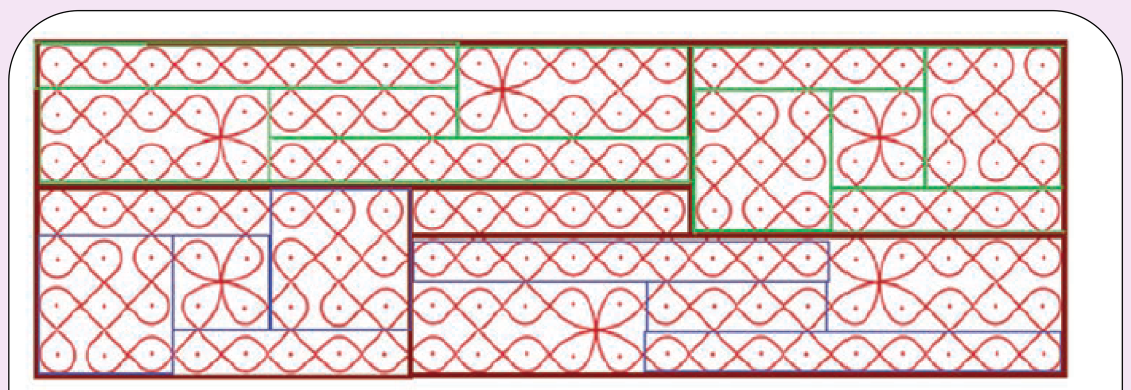


Figure 14 The π-kolam drawn on its skeleton diagram.

Super Golden Ratio Kolam

Now we will designing a new kolam using the kolam grammar discussed above. The mathematical constant that we are using is the super golden ratio. The super golden ratio is the ratio of the 2 consecutive terms of the Super Golden series. The first three terms are given to 1,1,1 and the subsequent term is the sum of the previous term and the term 2 places behind it. The series is as follows-
1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, 189, 277, 406, 595... and so on.
Mathematically, if we write the recurrence relation for this series, we get the following relation-

F(n) = F(n-1) + F(n-3) n>2 , n → Z

Here in this sequence, F(0) = F(1) = F(2) = 1.
The super golden ratio is denoted by the greek symbol psi (ψ), and is defined by the relation -

ψ = F(n)/F(n-1) n>2

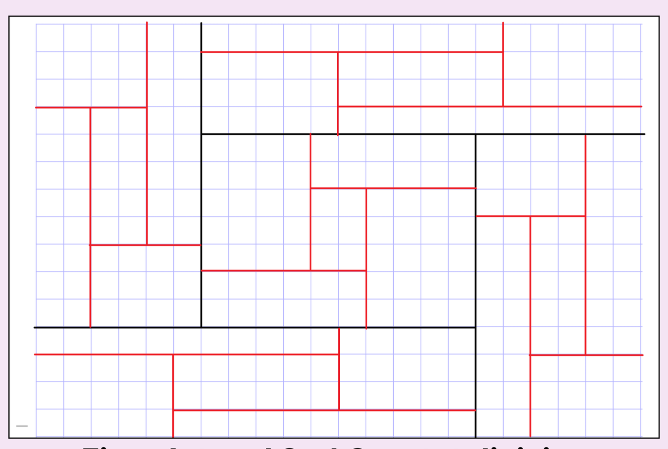
The super golden ratio is very similar to that of the golden ratio. Their exact values are solution for some polynomial. For the golden ratio we all know the equation, for the super golden ratio, the equation is given as-

x^3 = x^2+ 1

The only real solution to this equation is the value of the super golden ratio.

$$\psi = \frac{1+\sqrt[3]{\frac{29+3\sqrt{93}}{2}}+\sqrt[3]{\frac{29-3\sqrt{93}}{2}}}{3}$$

For determining the closest fraction for this constant, we need the continued fraction values for this sequence.The continued fraction array is non- repeating and infinite for this golden ratio, so we consider a few of the first values from the array.
The continued fraction array defined is -
→ 1, 2, 6, 1, 3, 5, 4, 22, 1, 1, 4, 1, 2, 84, 1
Using this and the continued fraction algorithm, we find the values of fractions that are close to the value of phi. we take the size of skeleton to be 22×15 as it is accurate upto 2 decimal places.
Now using the quartet rule discussed above, we find the quartets for this skeleton. The following are the quartets obtained and drawn on the skeleton.
Q1(7,4,11,15) and Q2(10,6,16,22)
Apllying quartet rule further, we get the following quartet pair-
7×11 → Q1(3,2,5,7) and Q2(2,4,6,10), 16×4 →Q1(2,1,3,4) and Q2(6,5,11,16), 6×11 →Q1(2,2,4,6) and Q2(5,3,8,11)
Once we have made the skeleton, we just need to fillin the spaces with primitives and connect them using splicing and unsplicing rules.
The following images show the stepwise construction.



Fig→ 1st and 2nd Quartet division

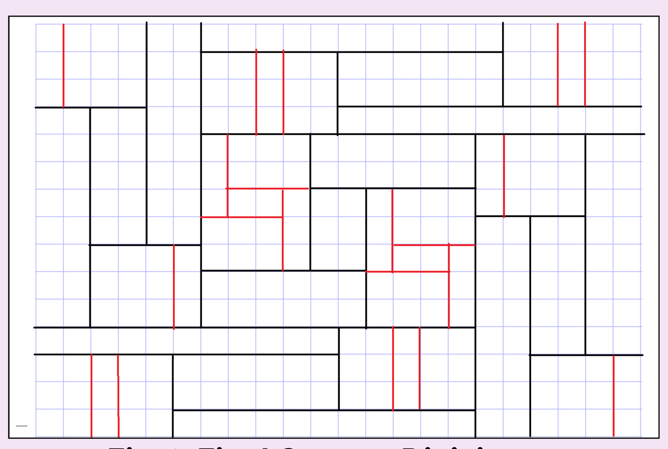
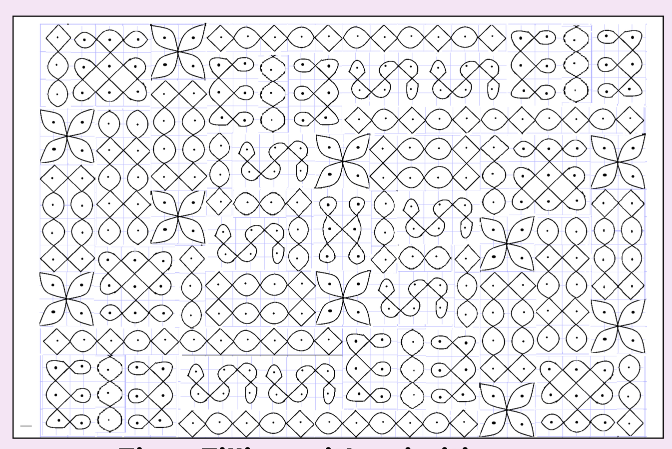


Fig → Final Quartet Division



Fig→ Filling with primitives

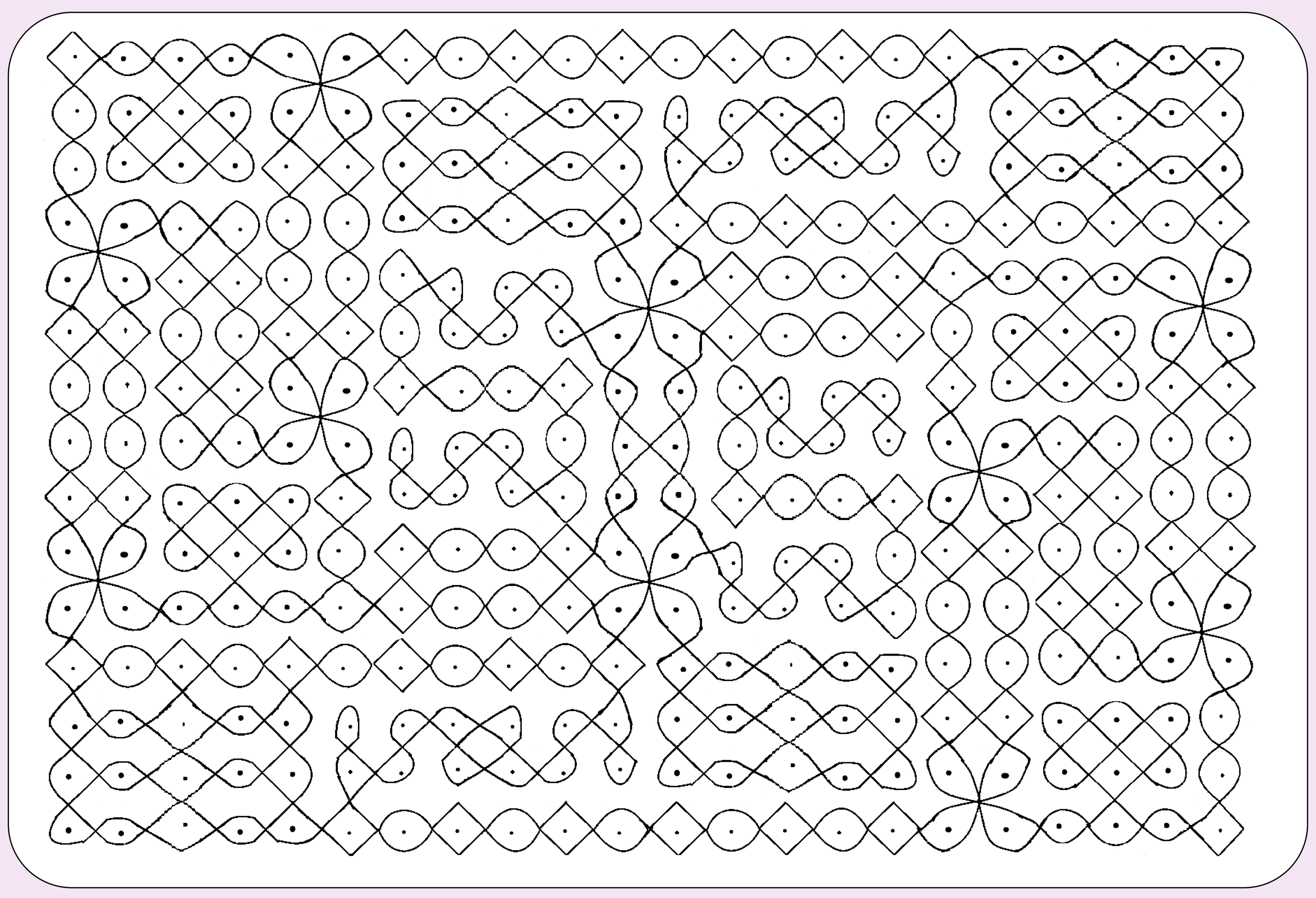


Fig → Final Kolam with Splicing

Note- Patterns for this kolam varies according to the choice of primitives. The position for splicing and unsplicing vary accordingly.

This type of kolam can fall under both Finite Matrix Kolam or Regular Matrix Kolam. This type of Mathematical kolam are often called Pulli kolam. These type of kolam have 2 way rotaional symmetry, and often fall under Finite matrix kolam. To this class belong all finite kolam patterns. It consists of distinct single patterns without any repetition or recursive element. Each pattern is an entity by itself and the dots for these belong to the class of finite matrix languages. These types of kolam fall under the category of Eulerian path, so iterative expansion is not possible in these types of kolam.

Such type of Kolam are generally created by hand and vary with sizes, so auotomted generation is not that easy for this. We are still trying figure out how to automate such type of kolam for a larger skeleton. We are exploring different softwares for this.